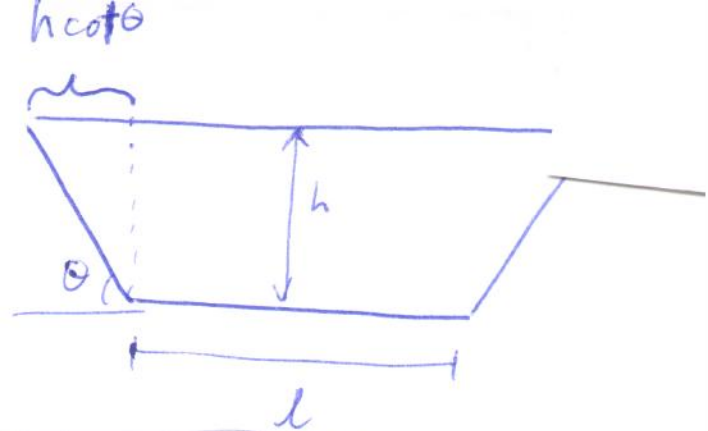


1) Weight:

$$\vec{F}_g = -gV\hat{k} = -g[l + h\cot\theta]hw\hat{k}$$



Pressure force:

bottom  $\vec{F}_g = -pA\hat{k} = \boxed{-gghlw\hat{k}}$

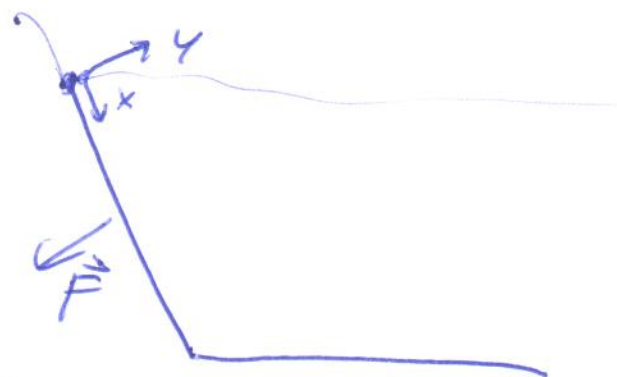
one side:

$$\vec{F}_p = \iint_S -p\hat{n}dS$$

$$\hat{n} = -\hat{j}$$

$$p = ggh$$

$$h = x \sin \theta$$

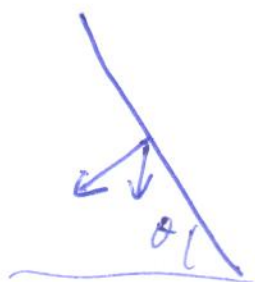


$$\vec{F}_p = \int_0^w \int_0^{w/\sin\theta} gg x \sin \theta \hat{j} dx dz$$

$$= w g g \sin \theta \left[ \frac{x^2}{2} \right]_0^{w/\sin\theta} = \frac{1}{2} w g g \frac{h^2}{\sin \theta}$$

$\Rightarrow$  global coordinate system

$$\text{vertical component} = -\frac{1}{2} w g g \frac{h^2}{\sin \theta} \cos \theta \hat{k} = -\frac{1}{2} w g g h^2 \cot \theta \hat{k}$$



total force = bottom + 2 sides

$$= -gghw[l + h\cot\theta]\hat{k}$$



$$2) \frac{dp}{dz} = \rho g$$

$$\rho = \frac{P}{RT}$$

$$g = -9.81 \text{ m/s}^2$$

2 choices :  $T = \text{constant}$

$$T = T_0 + Az$$

$$\Delta A = \frac{dT}{dz}, \text{ slope}$$

$T = \text{const}$

$$\frac{dp}{dz} = \frac{P}{RT} g \Rightarrow \int_{P_1}^{P_2} \frac{dp}{P} = \int_{z_1}^{z_2} \frac{g}{RT} dz$$

$$\ln\left(\frac{P_2}{P_1}\right) = \frac{g}{RT} (z_2 - z_1) \Rightarrow \boxed{\frac{P_2}{P_1} = e^{\frac{g(z_2 - z_1)}{RT}}}$$

$T = T_0 + Az$

$$\frac{dp}{dz} = \frac{P}{R(T_0 + Az)} g \Rightarrow \int_{P_1}^{P_2} \frac{dp}{P} = \int_{z_1}^{z_2} \frac{g}{R(T_0 + Az)} dz$$

$$\boxed{\ln\left(\frac{P_2}{P_1}\right) = \frac{g}{RA} \ln\left(\frac{T_0 + Az_2}{T_0 + Az_1}\right)}$$

$$\frac{P_2}{P_1} = \left(\frac{T_0 + Az_2}{T_0 + Az_1}\right)^{\frac{g}{RA}}$$

layer 1:  $T = T_0 + Az$

$P_0 = 101325 \text{ Pa}$      $A = -6.5 \times 10^{-3} \text{ K/m}$      $T_0 = 288 \text{ K}$

$$P = P_0 \left( \frac{288 \text{ K} - 6.5 \times 10^{-3} \text{ K/m } z}{288 \text{ K}} \right)^{\frac{-9.81 \text{ m/s}^2}{(287 \text{ J/kg K}) (-6.5 \times 10^{-3} \text{ K/m})}}$$

$P_1 = 101325 \text{ Pa} \rightarrow P|_{11 \text{ km}} = \boxed{22594 \text{ Pa}}$

layer 2:  $T = -56.5^\circ \text{C} = 216.5 \text{ K}$

$$P = P_2 e^{\left[ \frac{-9.81 \text{ m/s}^2 (z - 11000 \text{ m})}{287 \text{ J/kg K} (216.5 \text{ K})} \right]}$$

$P_2|_{20 \text{ km}} = \boxed{5456 \text{ Pa}}$

layer 3:  $T(z) = 216.5 \text{ K} + 1 \times 10^{-3} \text{ K/m} (z - 20000 \text{ m})$

$$P = P_3 \left( \frac{216.5 \text{ K} + 1 \times 10^{-3} \text{ K/m} (z - 20000 \text{ m})}{216.5 \text{ K}} \right)^{\frac{-9.81 \text{ m/s}^2}{(287 \text{ J/kg K}) (1 \times 10^{-3} \text{ K/m})}}$$

$P_4 = 5456 \text{ Pa} \left( \frac{278.6 \text{ K}}{216.5 \text{ K}} \right)^{\frac{-9.81 \text{ m/s}^2}{(287 \text{ J/kg K}) (1 \times 10^{-3} \text{ K/m})}} = \boxed{850 \text{ Pa}}$

layer 4:  $T(z) = 228.6 \text{ K} + 2.8 \times 10^{-3} \text{ K/m} (z - 32000 \text{ m})$

$$P = P_4 \left( \frac{228.6 \text{ K} + 2.8 \times 10^{-3} \text{ K/m} (z - 32000 \text{ m})}{228.6} \right)^{\frac{-9.81 \text{ m/s}^2}{287.5 \text{ J/kg K} (228.6 \text{ K})}}$$

$$P_5 = 450 \text{ Pa} \left( \frac{270.6}{228.6} \right)^{\frac{-9.81 \text{ m/s}^2}{(287.5 \text{ J/kg K}) (2.8 \times 10^{-3} \text{ K/m})}}$$

$$\boxed{P_5 = 108.5 \text{ Pa}}$$

layer 5:  $T = -2.5^\circ \text{C} = 270.6 \text{ K}$

$$P = P_5 e^{\left[ \frac{-9.81 \text{ m/s}^2 (z - 47000 \text{ m})}{287.5 \text{ J/kg K} (270.6 \text{ K})} \right]}$$

$$P_6 = 108.5 e^{\left( \frac{-9.81 \text{ m/s}^2 (4000 \text{ m})}{287.5 \text{ J/kg K} (270.6 \text{ K})} \right)}$$

$$\boxed{P_6 = 65.5 \text{ Pa}}$$

$$\text{layer 6: } T = 270.6 \text{ K} - 2.8 \times 10^{-3} \text{ K/m} (z - 51000 \text{ m})$$

$$P = P_6 \left( \frac{270.6 \text{ K} - 2.8 \times 10^{-3} \text{ K/m} (z - 51000 \text{ m})}{270.6 \text{ K}} \right)^{\frac{-9.81 \text{ m/s}^2}{(287 \text{ J/kg K}) (-2.8 \times 10^{-3} \text{ K/m})}}$$

$$P_z = 65.5 \text{ Pa} \left( \frac{214.6 \text{ K}}{270.6 \text{ K}} \right)^{\frac{-9.81 \text{ m/s}^2}{(287 \text{ J/kg K}) (-2.8 \times 10^{-3} \text{ K/m})}}$$

$$P_z = 3.9 \text{ Pa}$$

$$\text{layer 7: } T = 214.6 \text{ K} - 2 \times 10^{-3} \text{ K/m} (z - 71000 \text{ m})$$

$$P = P_7 \left( \frac{214.6 \text{ K} - 2 \times 10^{-3} \text{ K/m} (z - 71000 \text{ m})}{214.6 \text{ K}} \right)^{\frac{-9.81 \text{ m/s}^2}{(287 \text{ J/kg K}) (-2 \times 10^{-3} \text{ K/m})}}$$

$$P_8 = 3.9 \text{ Pa} \left( \frac{186.6 \text{ K}}{214.6 \text{ K}} \right)^{\frac{-9.81 \text{ m/s}^2}{(287 \text{ J/kg K}) (-2 \times 10^{-3} \text{ K/m})}}$$

$$P_9 = 0.4 \text{ Pa}$$

Rounding errors  
can pile up in this  
calculation. Your answer  
may differ a bit.

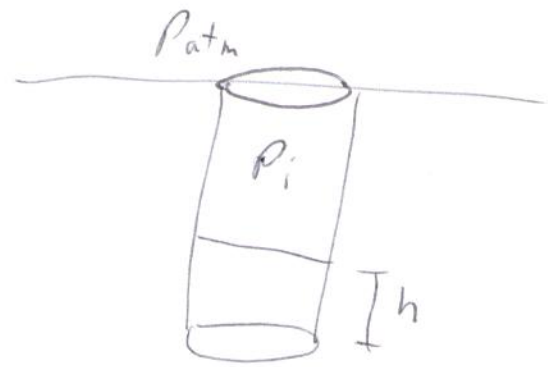
You can find  $\rho$  from the ideal gas law

$$3) a) P_i = P_{atm} + \rho_{H_2O} g (L-h)$$

Boyle's law:  $PV = \text{const}$

$$P_{atm} AL = P_i A (L-h)$$

$$P_i = \frac{P_{atm} AL}{A(L-h)}$$



$$P_{atm} + \rho_{H_2O} g (L-h) = \frac{P_{atm} L}{L-h} \Rightarrow \boxed{h = 0.082 \text{ m}}$$

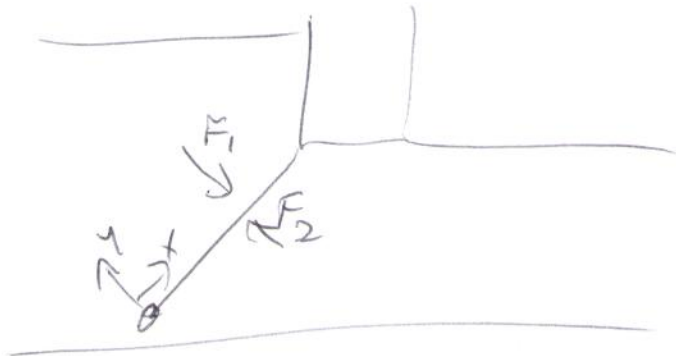
$$b) \vec{F} = P_i A \hat{k} - P_{atm} A \hat{k}$$

$$P_i = P_{atm} + \rho g (L-h) = 110334 \text{ Pa}$$

$$\vec{F} = (110334 \text{ Pa} - 101325 \text{ Pa}) (0.1 \text{ m}^2) = \boxed{901 \text{ N}}$$

$$4) \vec{dM} = \vec{dM}_1 + \vec{dM}_2$$

$$= \vec{r}_d \times (\vec{dF}_1 + \vec{dF}_2)$$



$$\vec{dF}_1 = -\rho_1 \hat{n}_1 dS \quad \vec{dF}_2 = -\rho_2 \hat{n}_2 dS$$

$$\hat{n}_1 = \hat{j}$$

$$\hat{n}_2 = -\hat{j}$$

$$\rho_1 = 39(8m - \frac{4}{5}x)$$

$$\rho_2 = 39(5m - \frac{4}{5}x)$$

$$\vec{r}_d = x\hat{i} + z\hat{k}$$

$$\vec{dM} = (x\hat{i} + z\hat{k}) \times [-39(8m - \frac{4}{5}x)\hat{j} + 39(5m - \frac{4}{5}x)\hat{j}] dx dz$$

$$= (x\hat{i} + z\hat{k}) \times [(-3m)39\hat{j}] dx dz$$

$$= (-3m)39 \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & 0 & z \\ 0 & 1 & 0 \end{vmatrix} dx dz = (-3m \cdot 39) \begin{bmatrix} -z\hat{i} \\ 0\hat{j} \\ x\hat{k} \end{bmatrix} dx dz$$

need

$$M_z = \int_0^{2m} \int_0^{5m} (-3m)39 x dx dz = -(3m)(2m)(1000 \frac{kg}{m^3})(9.81 \frac{m}{s^2})(\frac{25m^2}{2})$$

$$= -735750 \text{ Nm}$$

$$\vec{F} = \left( \frac{735750 \text{ Nm}}{5m} \right) \hat{j} = 147150 \text{ N } \hat{j}$$